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NUMERICAL MODELING OF THE PROCESS OF EMPLYING A TANK WITH A CRYOGENIC LIQUID

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Results of calculation of temperature and velocity fields in a cylindrical vessel with a cryogenic liquid (liquid hydrogen) upon its emptying are discussed. A problem is formulated for a viscous incompressible fluid at thermal boundary conditions of the first kind. Streamlines and isotherms for a variable fluid column are presented. Solutions are obtained for Rayleigh $Ra = 10^7 \ldots 10^{10}$ and Reynolds $Re = 10^3 \ldots 10^4$ numbers.

Construction of correct physical and adequate mathematical models of convective heat transfer processes in power-plant reservoirs under storage and consumption conditions of heat agents and in fuel tanks of aircraft is necessary for development of methods to increase the effectiveness of thermal-power systems and engines. In other words, theoretically based possibilities emerge to improve specific characteristics of converting thermal energy into performance power or its transfer.

In the present work we provide the results of a numerical solution of the problem of mixed free and forced convection in a liquid with its changing volume and thermal boundary conditions of the first kind. A problem is modeled of emptying of a cylindrical fuel tank in its upright position of a liquid component under highly nonisothermal conditions, e.g., with use of cryogenic fuel components (hydrogen or oxygen).

Liquid displacement from the tank with a prescribed flow rate causes forceds convection in the liquid volume. At the same time, the temperature difference between the liquid and the surrounding medium in the presence of the gravity field results in free convection of the liquid. In this case, the temperature field formed in the liquid with a change in its volume (tank emptying) determines, to a considerable degree, the working conditions of the tank as an element of the feed system of a power plant, i.e., the space-time variation of the temperature in the liquid volume is to be sought when estimating operation parameters, namely, the pressure or the flow rate when the liquid is drained from the tank.

In the adopted formulation of the problem, we consider an upright cylindrical vessel partially filled with a liquid or filled to height H and having a side generatrix prescribed by the equation $R = \varphi(z)$. The specific heat fluxes q_w, q_s, q_g are supplied to the side wall of the vessel, the free surface of the liquid (water table), and the flat bottom, respectively, or the temperatures are prescribed. The mass force is directed downwards parallel to the z-axis. The vessel bottom has a round hole with radius $r_0 = 0.1R$ through which a liquid can inflow or outflow at a constant flow rate. In this case, the vertical velocity V, which is constant with respect to the radius, is prescribed on the flat free surface of the liquid. The velocity changes in inverse proportion to the cross-sectional area of the tank in accordance with the flow rate.

The problem is solved in dimensionless form in a two-dimensional nonstationary formulation in cylindrical r, z-coordinates. To reduce the equations of conservation (of motion, energy, and discontinuity) to dimensionless form, the following relationships between dimensional and dimensionless quantities are introduced [1]:

$$\bar{u} = u/V ; \; \bar{v} = v/V ; \; \bar{\vartheta} = \frac{T - T_0}{T_w - T_0} ; \; \bar{p} = p/\rho V^2 ;$$

$$\bar{r} = r/R ; \; \bar{z} = z/R ; \; \mathrm{Ho} = V\tau/R , \tag{1}$$

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where T and T_0 is the liquid temperature at an arbitrary point of the volume and at the initial moment.

The quantities with a bar indicate the corresponding dimensionless variables. Henceforth the upper bar in the dimensionless equations is omitted.

Upon introducing dimensionless variables of form (1) and (2), the equations of conservation, the initial and boundary conditions will be written as

$$\frac{\partial u}{\partial \mathrm{Ho}} + u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} = -\frac{\partial p'}{\partial r} + \frac{1}{\mathrm{Re}}\left(\frac{\partial u^2}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\frac{\partial^2 u}{\partial z^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial \mathrm{Ho}} + u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z} = -\frac{\partial p'}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial v^2}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right) + \frac{\mathrm{Gr}_R}{\mathrm{Re}^2}\vartheta, \tag{3}$$

$$\frac{\partial u}{\partial r} + \frac{u}{r}\frac{\partial v}{\partial z} = 0, \tag{4}$$

$$\frac{\partial \vartheta}{\partial \mathrm{Ho}} + u\frac{\partial \vartheta}{\partial r} + v\frac{\partial \vartheta}{\partial z} = \frac{1}{\mathrm{Pr\,Re}}\left(\frac{\partial \vartheta^2}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta}{\partial r} + \frac{\partial^2 \vartheta}{\partial z^2}\right). \tag{5}$$

The similarity numbers in (3) and (5) are the Grashof number $\mathrm{Gr} = g\beta(T_w - T_0)R^3/\nu^2$; the Reynolds number $\mathrm{Re} = VR/\nu$; the Prandtl number $\mathrm{Pr} = \nu/a$; dimensionless time (homochronocity) $\mathrm{Ho} = Vt/R$.

In writing the boundary conditions for the equation of motion, use is made of the ordinary boundary conditions for a viscous fluid, i.e., adhesion on solid walls and flow axisymmetry, are adopted.

The initial conditions are prescribed in form of the known functions u^0, v^0, ϑ^0. The temperature field was assumed to be either uniform or have vertical stratification after prolonged storage in accordance with the linear law $\vartheta = 0$ near the bottom and $\vartheta = 1$ near the free surface.

The liquid at the initial moment was assumed to be at rest: $u^0 = v^0 = 0$.

For numerical solution the initial system of Eqs. (2)-(5) and the boundary conditions were transformed by introducing the eddy function ω and the stream function ψ [1] satisfying the relations

$$u = -\frac{\partial \psi}{\partial z}, \quad v = \frac{\partial \psi}{\partial r}, \quad \omega = \frac{1}{r}\left(\frac{\partial v}{\partial r} - \frac{\partial u}{\partial z}\right). \tag{6}$$

The problem discussed has some specific features to be allowed for in its solution, namely, that the velocity and temperature fields undergo the largest changes near the side wall, where a boundary layer is formed. For this, one more transformation of the coordinates was made:

$$r = \frac{1}{n_1}\ln\left(1 + (e^{n_1} - 1)\,r_1\right), \quad z = \frac{1}{m_1}\ln\left(1 + (e^{m_1} - 1)\,z_1\right). \tag{7}$$

In so doing, the integration region was deformed in the direction of the $0r_2$- and $0z_2$-axes. The degree of contraction is controlled by the parameters n_1 and m_1, the values of which are determined by the uniformity of the integration network: $n_1 = m_1 = 0.3$.

The algorithm is based on the difference schemes proposed by V. I. Polezhaev for solving the equations of a viscous incompressible fluid [1, 2]. To approximate the differential operators, central differences and an approximation by directional differences [2] were employed. Use of the directional differences allowed an increase in the upper limit of the Rayleigh and Reynolds numbers.

Approximation of the boundary conditions of the vortex equation was made using approximate boundary conditions, which can be obtained by either Taylor series expansion of the stream function ψ near the boundary

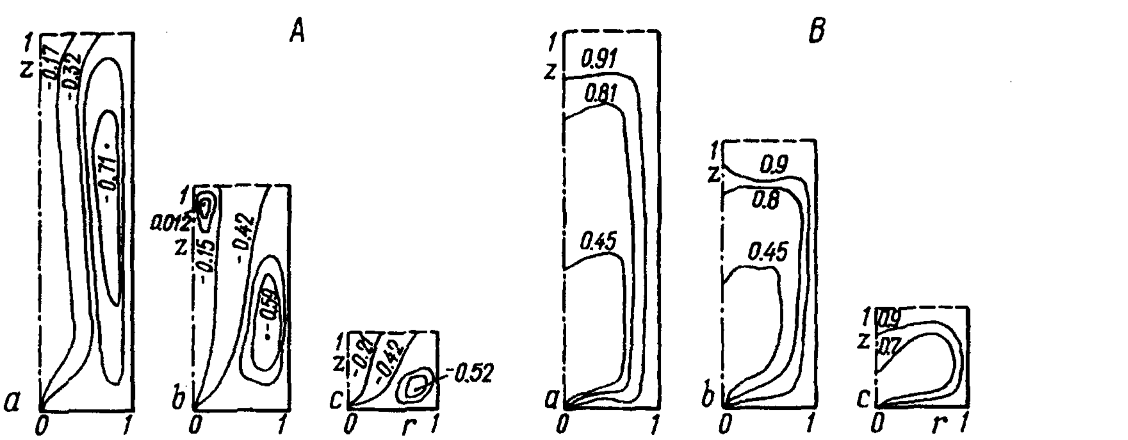

Fig. 1. Streamlines (A) and isotherms (B) for the case of homogeneous liquid discharge (Re = 5000; Ra = 10^{10}): a) Ho = 1; b) 3; c) 5.

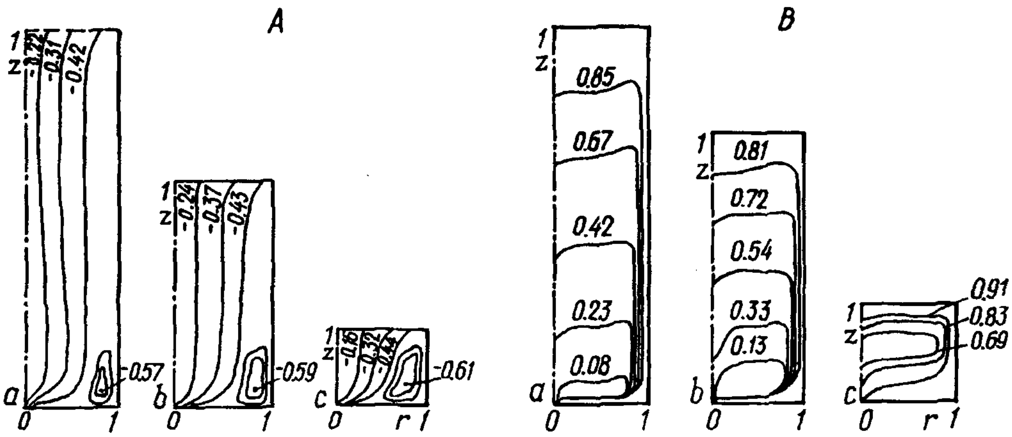

Fig. 2. Streamlines (A) and isotherms (B) for the case of nonisothermal liquid discharge (Re, Ra, and Ho are the same as in Fig. 1.).

or using the procedure [1] for solving the equations of a viscous incompressible fluid in terms of the variables $\psi - \omega - \vartheta$.

Following the model described above, we composed a program in FORTRAN. Calculations were made for Rayleigh $10^7 < \mathrm{Ra} \leq 10^{10}$ and Reynolds $1000 \leq \mathrm{Re} \leq 10{,}000$ numbers at Pr = 1 (Pr ≈ 1 corresponds to liquid hydrogen in the saturation state). Over the drain hole, a constant velocity and intake radius $r_{\mathrm{in}} = 0.1R$ were prescribed. The initial height of filling was $l_0 = 6R$, the final height – $l_0 = R$.

Figure 1 shows a structure of mixed convection in the case of vessel emptying and temperature fields for an initially homogeneous liquid. Note that an increase of the rate of vessel emptying considerably changes the macrovorticity structure of the flow in the volume: from a marked influence of the macrovortex caused by the free lifting convection of the fluid near the wall at low Reynolds numbers (Re ≤ 2000, $\mathrm{Ra}_R = 10^{10}$) to a decrease in the region of its spreading, and the intensity at Re ≥ 8000, $\mathrm{Ra}_R = 10^{10}$. The center of this vortex is displaced downward along the wall.

Owing to the appearance (as in the case of vessel filling) of a more heated liquid (as compared to its basic mass) over the vessel axis, an inverse vortex is formed there. We estimated the boundary of its emergence near the axis and found that it occurs at $\mathrm{Ra}/\mathrm{Re}^2 \geq 10^{2.7}$.

A comparison of the dimensionless isotherms for the regimes considered reveals that an increase in the fluid flow rate markedly decreases temperature stratification with respect to the height of the fluid volume as compared to the regime of filling at a lower flow rate.

Figure 2 shows the flow picture and the temperature field for the case of nonisothermal liquid drain. The drain of the stratified liquid is characterized by a decrease in the free-convection vortex as compared to the homogeneous liquid discharge. Moreover, in the case of the nonisothermal liquid drain no reverse vortices are seen over the axis.

It is established that at first heat transfer is mainly determined by the Reynolds number. Then the developing free-convective flow begins to exert a pronounced influence. At the end of emptying at small heights of filling ($l_{min}/R = 1$) the influence of the Reynolds number is again considerable (the forced flow suppresses the free-convective vortex). For calculating the Nusselt number (drain of the homogeneous liquid) the following generalized relation is obtained

$$\overline{\mathrm{Nu}}_R = 10^{-1.15}\,\mathrm{Ra}^{0.05}\,\mathrm{Re}^{0.7}\,.$$

The developed model permits one to make variant calculations of convective heat transfer in a variable volume of liquid under nonisothermal conditions and to obtain substantiated recommendations for optimization of the regimes of filling and emptying of vessels and tanks.

NOTATION

a, thermal diffusivity; g_0 gravity acceleration; l_0, current height of tank filling; p, pressure; q, specific heat flux; R, maximum radius of the tank; r, radial coordinate; r_{in}, intake radius; T_0, initial temperature; T_w, wall temperature; V, velocity scale; u, v, projections of the velocity vector on the $0r$, $0z$-coordinate axes; z, vertical coordinate; β, volume expansion coefficient; ρ, density; λ, thermal conductivity; ν, kinematic viscosity; ψ, stream function; ω, eddy function; $\mathrm{Gr} = g\beta(T_w - T_0)R^3/\nu^2$, Grashof number; $\mathrm{Re} = VR/\nu$, Reynolds number; $\mathrm{Pr} = \nu/a$, Prandtl number; $\mathrm{Ho} = Vt/R$, dimensionless time (homochromaticity number); $\vartheta = (T - T_0)/(T_w - T_0)$, dimensionless temperature; $\bar{u} = u/V$, dimensionless horizontal velocity; $\bar{v} = v/V$, dimensionless vertical velocity; $\overline{\mathrm{Nu}}_R = \alpha R/\lambda$, Nusselt number.

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